Signal of neutrinoless double beta decay, neutrino spectrum and oscillation scenarios

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Abstract: The lower and upper bounds on the neutrinoless double beta ($0\nu2\beta$) decay rate are obtained, as functions of the parameters of neutrino oscillations and of the lightest neutrino mass. The constraints on these parameters from the search for the $0\nu2\beta$ transition, as well as from the interpretation of solar and atmospheric neutrino data in terms of oscillations, can be conveniently represented in one unitarity triangle. This representation helps to clarify the cases when the $0\nu2\beta$ rate is small; the crucial dependence on the scenarios assumed for solar neutrino oscillations and on the neutrino spectrum is emphasized. We consider hierarchical and non-hierarchical neutrino spectra, and discuss their interest in view of future searches of the $0\nu2\beta$ decay.

Keywords: Neutrino Physics, Solar and Atmospheric Neutrinos.

Contents

1. Informations on neutrino parameters

1.1 Massive neutrinos and $0\nu2\beta$ decay

Atmospheric neutrino data can be interpreted in terms of a dominant $\nu_\mu - \nu_\tau$ oscillation channel, although a sub-dominant channel $\nu_\mu - \nu_{\rm e}$ is not excluded [1]. The latter may be due to a ν_e component of the heaviest (lightest) neutrino state ν_3 (ν_1) for spectra with "normal" ("inverted") hierarchy–our definition of "hierarchy" is discussed in section 3. Several possibilities are open for the interpretation of the solar neutrino data, depending on the frequencies of oscillation and mixings.

Hence, the indications for massive neutrinos are strong. However, there is quite a limited knowledge on the *neutrino mass spectrum itself*, and particularly on the lightest neutrino mass. The search for $0\nu2\beta$ decay can shed light on this important issue. The bound of 0.2 eV obtained [2] on the parameter

$$\mathcal{M}_{\rm ee} = |\sum_i U^2_{{\rm e}i}\ m_i| \tag{1.1}$$

is sensibly smaller than the mass scales probed by present studies of β-decay, or those inferred in cosmology [3]. In eq. (1.1), the non-negative quantities m_i, $i = 1, 2, 3...N$ are the neutrino masses ($m_{i+1} \geq m_i$); the complex quantities $U_{\ell i}$, $\ell = \mathrm{e}, \mu, \tau...$, are the elements of the mixing matrix, which relates the flavor eigenstates to the mass eigenstates: $\nu_\ell(x) = \sum_i U_{\ell i}\, \nu_i(x)$. Hence, $\mathcal{M}_{\mathrm{ee}}$ can be thought of as (the absolute value of) the ee$-$entry of the neutrino mass matrix. Let us recall that, beside the $(N-1)(N-2)/2$ phases relevant to neutrino oscillations, there are still $N-1$ physical phases in the lepton sector that have no analogy in the quark sector, and arise from the Majorana structure of the neutrino mass matrix. Notice that both the amplitudes *and* the phases of the elements of the mixing matrix $U_{\mathrm{e}i}$ are relevant in determining the size of $\mathcal{M}_{\mathrm{ee}}$.

1.2 Extremal values of $\mathcal{M}_{\mathrm{ee}}$ for $0\nu2\beta$ decay

We obtain in this section the extremal values of $\mathcal{M}_{\mathrm{ee}}$ under arbitrary variations of the phases, keeping fixed the neutrino masses m_i and the "mixing elements"[1] $|U^2_{\mathrm{e}i}|$. The maximum value of $\mathcal{M}_{\mathrm{ee}}$ is simply:

$$\mathcal{M}_{\mathrm{ee}}^{max} = \sum_i |U^2_{\mathrm{e}i}|\; m_i. \tag{1.2}$$

The minimum value can be written as:

$$\mathcal{M}_{\mathrm{ee}}^{min} = \max\{\; 2\; |U^2_{\mathrm{e}i}|\; m_i - \mathcal{M}_{\mathrm{ee}}^{max},\quad 0\;\}. \tag{1.3}$$

To demonstrate this formula, let us consider the absolute value of the sum of three complex numbers: $r = |z_1 + z_2 + z_3|$. We want to minimize r by keeping fixed $|z_i|$, namely, by varying the phases. Let us define the quantities $r_{1,2,3}$ and $q_{1,2,3}$ as: $r_1 = |z_1| - |z_2| - |z_3|$, $q_1 = |z_1| - |z_2 + z_3|$, and similar eqs., but permuting the indices for $r_{2,3}$ and $q_{2,3}$. Notice that *at most* one of the r_i's is positive. Assuming that $r_1 > 0$, it is simple to show that $r^{min} = r_1$; in fact, using twice the Schwartz inequality, we get $r \geq |q_1| = q_1 \geq r_1$. Similar considerations if $r_2 > 0$, or $r_3 > 0$. The last case has $r_i \leq 0$ for $i = 1, 2, 3$. If one of the r_i's is zero, then $r^{min} = 0$, hence we need to consider the case when $r_i < 0$ for all i's. In this case, the quantity q_1 goes from negative, when the phases of z_2 and z_3 are equal, to positive, when these phases are opposite. By continuity, a phase choice exists such that $q_1 = 0$. Since by proper choice of the phase of z_1 we can get $r = |q_1|$ we conclude that, again, $r^{min} = 0$. In conclusion, the general case is covered by the formula: $r^{min} = \max\{r_i,\; 0\}$. This is equivalent to eq. (1.3), after noticing that $r_i = 2|z_i| - \sum_{i=1}^{3} |z_i|$. The generalization of these results to N neutrinos is quite simple: Just limit the sum in eq. (1.2) to $N = 3$. However, we will be concerned only with the case of three neutrinos in the rest of the work.

[1] In the following, we will always refer with the term "mixing elements" to the absolute value of the elements of the mixing matrix.

The previous two equations give the extremal values of $\mathcal{M}_{\rm ee}$, once the neutrino spectrum *and* the mixing elements are known. Such extremal values are important, being independent of the complex phases. The information we get from the experimental upper bound is $\mathcal{M}_{\rm ee}^{bound} \geq \mathcal{M}_{\rm ee}^{min}$; the informations we could get from a positive signal, instead, is $\mathcal{M}_{\rm ee}^{signal} \in [\mathcal{M}_{\rm ee}^{min},\ \mathcal{M}_{\rm ee}^{max}]$. In the following it will be shown how to use and represent $\mathcal{M}_{\rm ee}^{min}$ and $\mathcal{M}_{\rm ee}^{max}$, and what we can learn on them assuming specific neutrino spectra, and scenarios of neutrino oscillations.

2. Representation of $\mathcal{M}_{\rm ee}^{min}$ and $\mathcal{M}_{\rm ee}^{max}$

We introduce and discuss in this section a graphical representation of the values of $\mathcal{M}_{\rm ee}^{min}$ and $\mathcal{M}_{\rm ee}^{max}$. For this purpose we will make reference to fig. 1, where the representation of $\mathcal{M}_{\rm ee}^{min}$ is displayed, for an illustrative choice of the neutrino spectrum: $m_3 = 2\, m_2$ and $m_2 = 2\, m_1$. In order to fix the ideas, we point out from the beginning the two essential features of fig. 1: (1) the value of $\mathcal{M}_{\rm ee}$ at the vertices, namely the masses of the neutrinos m_i; (2) the position of the inner triangle (also determined by the masses of the neutrinos).

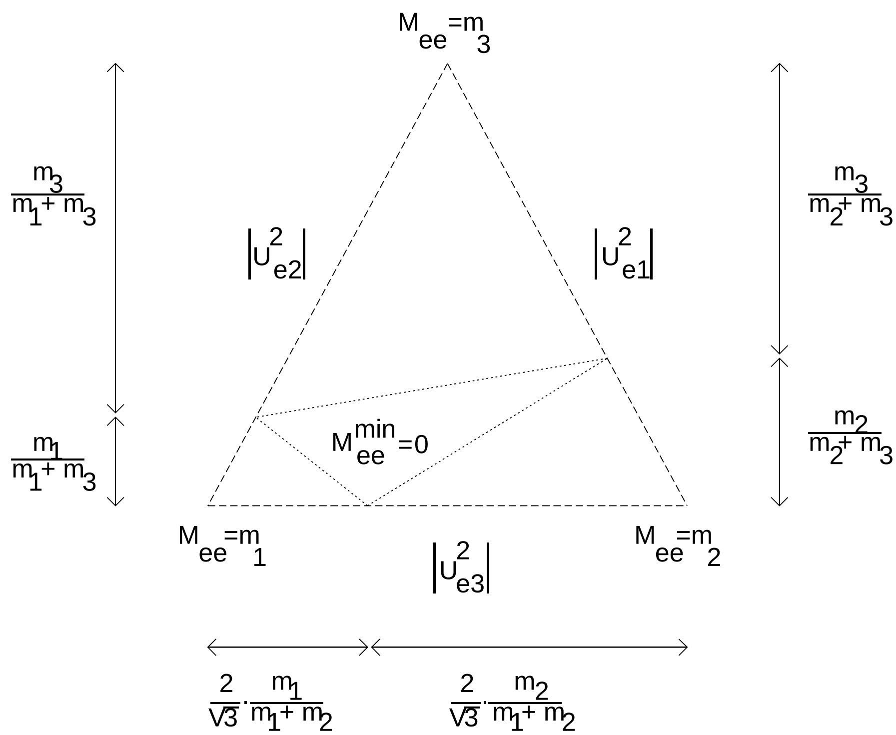

Figure 1: Representation of the minimum value of $\mathcal{M}_{\rm ee}$, eq. (1.3), in the unitarity triangle. For a given internal point, the distance from the side labelled by $|U_{ei}^2|$ represents the size of the corresponding mixing element. The inner triangle encloses the region where $\mathcal{M}_{\rm ee}^{min} = 0$. The position of the vertices of the inner triangle relative to the vertices of the unitarity triangle (corresponding to eq. (2.1)) is indicated by the arrows with labels.

Let us begin by recalling some basic facts. The three mixing elements $|U_{ei}^2|$ are constrained by the unitarity condition $\sum_i |U_{\mathrm{e}i}^2| = 1$. This condition can be represented by using the inner region of one equilateral triangle with unit height, where the distance from the i^{th} side represents the value of $|U_{\mathrm{e}i}^2|$, see fig. 1 (this triangle was first used in [4], to analyze solar neutrino oscillations). To exemplify the use of the triangle, let us consider two special cases: (a) When ν_e is an equal admixture of the three mass eigenstates, we have $|U_{\mathrm{e}i}^2| = 1/3$. This point is represented by the barycentre of the equilateral triangle of fig. 1. (b) When ν_e coincides with the mass eigenstate ν_1, we have $|U_{\mathrm{e}1}^2| = 1$, and the other two mixing elements are zero. This point is represented by the 1^{st}-vertex (by definition, the 1^{st} vertex is opposite to the 1^{st} side, denoted with the label $|U_{\mathrm{e}1}^2|$ in fig. 1, *etc.*).

From eq. (1.3), $\mathcal{M}_{\mathrm{ee}}^{min}$ is *zero* in the inner triangular region represented in fig. 1. The vertices of this inner triangle are given by:

$$|U_{\mathrm{e}1}^2|/|U_{\mathrm{e}2}^2| = m_2/m_1 \text{ when } |U_{\mathrm{e}3}^2| = 0, \tag{2.1}$$

and by the two additional equations obtained by the replacement $3 \leftrightarrow 1$, and $3 \leftrightarrow 2$. The condition $|U_{\mathrm{e}3}^2| = 0$ in eq. (2.1) tells us that we are on the 3^{rd} (lower) side of the unitarity triangle of fig. 1.

At the i^{th} vertex of the unitarity triangle $\mathcal{M}_{\mathrm{ee}}^{min} = \mathcal{M}_{\mathrm{ee}} = m_i$, as is clear from eq. (1.3), and as illustrated in fig. 1. The value of $\mathcal{M}_{\mathrm{ee}}^{min}$ decreases linearly when moving from one vertex toward the inner triangle. In fact, $\mathcal{M}_{\mathrm{ee}}^{min}$ is non-zero only close to the vertices of the unitarity triangle (assuming $m_1 > 0$). This concludes the illustration of fig. 1.

The unitarity triangle can also be used to represent the maximum possible value $\mathcal{M}_{\mathrm{ee}}$. Quite simply, $\mathcal{M}_{\mathrm{ee}}^{max}$ is the function of the mixing elements $|U_{\mathrm{e}i}^2|$ that interpolates linearly among the values $\mathcal{M}_{\mathrm{ee}} = m_i$ taken at the vertices of the unitarity triangle, as clear from eq. (1.2). However, since $\mathcal{M}_{\mathrm{ee}}^{max}$ is just the sum of positive contributions (eq. (1.2)), the analysis of $\mathcal{M}_{\mathrm{ee}}^{max}$ is nearly trivial.

3. Phenomenology of oscillations and $0\nu2\beta$

We discuss now the $0\nu2\beta$ signal assuming some specific spectra, and scenarios of oscillation, using the graphical representation introduced above. We take advantage of the indications from atmospheric and solar neutrinos, that can be accounted in terms of two different frequencies of neutrino oscillations, related to the mass differences squared Δm^2_{atm} and $\Delta m^2_{\odot}$ ($\Delta m^2_{atm} \gg \Delta m^2_{\odot}$). We consider the following three cases:

- Case [$\mathcal{N}$]: "normal" hierarchy, $m_1 \ll (\Delta m^2_{atm})^{1/2}$;
- Case [$\mathcal{I}$]: "inverted" hierarchy, $m_1 \ll (\Delta m^2_{atm})^{1/2}$;
- Case [$\mathcal{D}$]: "normal" and "inverted" hierarchies, $m_1 \gg (\Delta m^2_{atm})^{1/2}$;

from these cases, it will be easy to understand also the "intermediate" situations when $m_1 \sim (\Delta m^2_{atm})^{1/2}$. With the term "hierarchy" (either "normal" or "inverted") we refer to *the mass differences squared* (see eqs. (3.1) and (3.4) below)[2]. We assume that the electronic admixture in atmospheric neutrinos is sub-dominant [1], and use for the mass splittings Δm^2_{atm} and $\Delta m^2_{\odot}$ the values suggested by the phenomenology. For solar neutrino solutions we use the terminology of [5], that we will recall in the following. A similar study has been performed in reference [6], with the goal to extract informations on the mixing angles, knowing $\mathcal{M}_{\rm ee}$ and the neutrino spectrum. For other recent works oriented toward the phenomenology, see [7].

3.1 Case [$\mathcal{N}$]: "normal" hierarchy, $m_1 \ll (\Delta m^2_{atm})^{1/2}$

What is the expected value of $\mathcal{M}_{\rm ee}$ for a neutrino spectrum with "normal" hierarchy:

$$m_3^2 - m_2^2 = \Delta m^2_{atm} \gg m_2^2 - m_1^2 = \Delta m^2_{\odot}, \tag{3.1}$$

assuming, to begin with, that m_1 is negligible? For the values of $\Delta m^2_{\odot}$ suggested by the MSW [8] small mixing angle solution of the solar neutrino problem (SMA) or vacuum oscillation (VO), the only important contribution to $0\nu2\beta$ decay rate comes from the heaviest eigenstate: $\mathcal{M}_{\rm ee} \approx |U^2_{\rm e3}| m_3$. It is *possible* to have a comparable contribution from the second eigenstate assuming MSW solutions of the solar neutrino problem with large mixing angle (LMA) $\delta\mathcal{M}_{\rm ee}|_{\odot} = |U^2_{\rm e2}|\ m_2 \approx 4 \times 10^{-3}$ eV (using $\Delta m^2_{\odot} \approx 10^{-4}$ eV^2 and $|U^2_{\rm e2}| \approx 0.4$). This is of the same size of the contribution from the heaviest eigenstate, $\delta\mathcal{M}_{\rm ee}|_{atm} = |U^2_{\rm e3}|\ m_3$, if $|U^2_{\rm e3}| \approx 0.1$ and $\Delta m^2_{atm} \approx 2 \times 10^{-3}$ eV^2. We conclude that, if future experiments searching for the $0\nu2\beta$ transition will prove that

$$\mathcal{M}_{\rm ee} > 10^{-2} \text{ eV}, \tag{3.2}$$

the hypothesis of a spectrum with "normal" hierarchy and very small m_1 will be disfavoured [9][3].

The function $\mathcal{M}^{min}_{\rm ee}$ is represented in fig. 2 for two different values of $\Delta m^2_{\odot}$: 10^{-4} eV^2 in the 1^{st} plot, and 10^{-5} eV^2 in the 2^{nd} (we assumed $\Delta m^2_{atm} = 2 \times 10^{-3}$ eV^2). Notice that assuming $m_1 = 0$ the inner triangle of fig. 1 degenerates into a line (for much smaller values of $\Delta m^2_{\odot}$, say for VO, the line practically coincides with the side $U_{\rm e3} = 0$). Recalling that the inner triangle corresponds to the region where $\mathcal{M}^{min}_{\rm ee} = 0$, we appreciate from fig. 2 the crucial dependence on the parameter $|U^2_{\rm e3}|$ of the $0\nu2\beta$ transition rate.

[2]In order to simplify the connection with the phenomenology, we use a definition of "hierarchy" that is relevant to neutrino oscillations, which involves *just* the mass differences squared. Notice that sometimes in the literature, "hierarchy" is used in reference to the neutrino spectrum itself.

[3]Alternatively, one should postulate a different origin of the $0\nu2\beta$ decay.

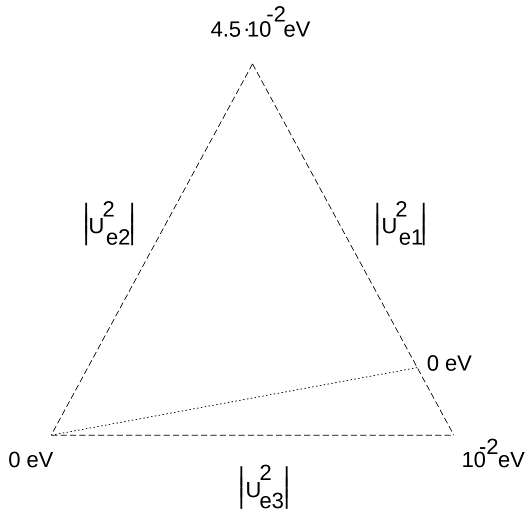

Figure 2: Same as fig. 1, for spectra with $m_1 = 0$ and "normal" hierarchy. For "inverted" hierarchy, the value at the 2^{nd} vertex increases from the value 10^{-2} in the first plot (3×10^{-3} in the second plot) up to $\approx 4.5 \times 10^{-2}$ eV, and the internal line approaches closely the bisector $|U^2_{\rm e2}| = |U^2_{\rm e3}|$.

Let us increase now the size of m_1, keeping $m_1 \ll (\Delta m^2_{atm})^{1/2} \approx m_3$. The (degenerate) inner triangle in fig. 2 becomes an *obtuse* isosceles triangle when $m_1 \approx m_2 \gtrsim (\Delta m^2_\odot)^{1/2}$; the base being parallel to the 3^{rd} side, where $U_{\rm e3} = 0$. A complete suppression of the $0\nu2\beta$ transition can take place for those solutions of the solar neutrino problem that fall in this inner triangle, and for this reason, the most important conclusion is unchanged: The size of $|U^2_{\rm e3}|$ is very important in determining whether the case $\mathcal{M}^{min}_{\rm ee} = 0$ is possible or not. More precisely, this mixing element has to be compared with the height of the triangle, $\sim m_1/m_3$ (see fig. 1). Incidentally, we notice the simple formula

$$\mathcal{M}_{\rm ee} \approx |m_1 + |U^2_{\rm e3}|\ m_3\ e^{i\varphi}| \quad \text{ where } \varphi = \arg[\,U^2_{\rm e3}/U^2_{\rm e1}\,] \tag{3.3}$$

valid for the SMA case, which illustrates that $\mathcal{M}_{\rm ee} \approx 0$ is possible when $m_1/m_3 \approx |U^2_{\rm e3}|$ ($m_3 \approx (\Delta m^2_{atm})^{1/2}$ in present hypotheses) and the phases of $U^2_{\rm e3}$ and $U^2_{\rm e1}$ are opposite.

3.2 Case [$\mathcal{I}$]: "inverted" hierarchy, $m_1 \ll (\Delta m^2_{atm})^{1/2}$

Let us assume a spectrum with "inverted" hierarchy, namely

$$m_3^2 - m_2^2 = \Delta m^2_\odot \ll m_2^2 - m_1^2 = \Delta m^2_{atm}, \tag{3.4}$$

and suppose, to begin with, that m_1 is negligible. In this case, since the sub-dominant mixing element is $|U^2_{\rm e1}|$, we can obtain large maximum values [12]:

$$\mathcal{M}^{max}_{\rm ee} \approx (\Delta m^2_{atm})^{1/2} = (3 \text{ to } 9) \times 10^{-2} \text{ eV}. \tag{3.5}$$

This could be close to the present bound [2], if also the nuclear matrix elements take the highest values allowed by present uncertainties, $\sim 2-3$ [11].

In these hypotheses, $\mathcal{M}_{\rm ee}^{min}$ can be (close to) zero only if $|U_{\rm e2}^2|$ is very close to $|U_{\rm e3}^2|$; the contribution from $|U_{\rm e1}^2|$ being irrelevant. In a graphical representation like in fig. 2, this corresponds to the fact that the inner triangle almost coincides with the bisector $|U_{\rm e2}^2| = |U_{\rm e3}^2|$ (the "small" mixing element $|U_{\rm e1}^2|$ is represented by the distance from the 1^{st}–right–side).

Let us increase the size of m_1, keeping $m_1 \ll (\Delta m^2_{atm})^{1/2} \approx m_3$. The inner triangle is, in this assumption, *acute* isosceles, the base being parallel to the side $U_{\rm e1} = 0$, and with length $\sim m_1/m_3 \times 2/\sqrt{3}$. Hence, only those solutions of the solar neutrino problem which have almost maximal mixing angles (VO, averaged oscillations and perhaps LMA) fall in the region where the $0\nu2\beta$ transition rate may be strongly suppressed. In the case of SMA, since $|U_{\rm e3}^2|$ is small by assumption (and $|U_{\rm e1}^2|$ is not large) we have simply:

$$\mathcal{M}_{\rm ee} \approx m_2 \approx (m_1^2 + \Delta m^2_{atm})^{1/2}. \tag{3.6}$$

Hence, $\mathcal{M}_{\rm ee} \approx 0$ is impossible if the SMA solution is correct. Quite generally, in the case of "inverted" hierarchy, it is less likely that $\mathcal{M}_{\rm ee}^{min}$ is zero.

3.3 Case [$\mathcal{D}$]: "nearly degenerate" spectrum, $m_1 \gg (\Delta m^2_{atm})^{1/2}$

Largest values of $\mathcal{M}_{\rm ee}$ (up to the experimental bound) can be taken for a "nearly degenerate" neutrino spectrum [10, 13]. The maximum value is simply $\mathcal{M}_{\rm ee}^{max} = m_1 + \mathcal{O}(\Delta m^2/m_1)$, m_1 playing the role of mass spectrum offset.

The corresponding minimum value, $\mathcal{M}_{\rm ee}^{min}/m_1 = \max\{2|U_{{\rm e}i}^2| - 1,\ 0\}$ is represented in fig. 3 assuming "normal" hierarchy of the mass differences (eq. (3.1)); $\mathcal{O}(\Delta m^2/m_1^2)$ terms have been neglected. From this figure it is visible that, to interpret properly the results of $0\nu2\beta$ decay studies (and possibly, to exclude the inner region in the 1^{st} plot, the one where $\mathcal{M}_{\rm ee} \ll m_1$ is *possible*) we need precise information on the mixing elements. This requires distinguishing among oscillation scenarios. The plots also illustrate the importance to quantify the size of $|U_{\rm e3}^2|$ [10, 13], [1]. Similar considerations apply when the mass differences have "inverted" hierarchy, eq. (3.4) with $|U_{\rm e1}^2|$ playing the role of $|U_{\rm e3}^2|$. Notice in particular that with approximate mass degeneracy the role of the sub-dominant mixing is almost the same for "normal" and "'inverted" hierarchy; this should be contrasted with the conclusions for the cases [$\mathcal{N}$] and [$\mathcal{I}$], when $m_1 \ll (\Delta m^2_{atm})^{1/2}$.

In the particular case of SMA solution, eq. (3.3) is still valid, with $m_1 \approx m_3$ (and $U_{\rm e3} \to U_{\rm e1}$ for "inverted" hierarchy); hence, up to sub-dominant mixing terms $\mathcal{M}_{\rm ee} \approx \mathcal{M}_{\rm ee}^{max} \approx m_1$, and a complete cancellation is impossible.

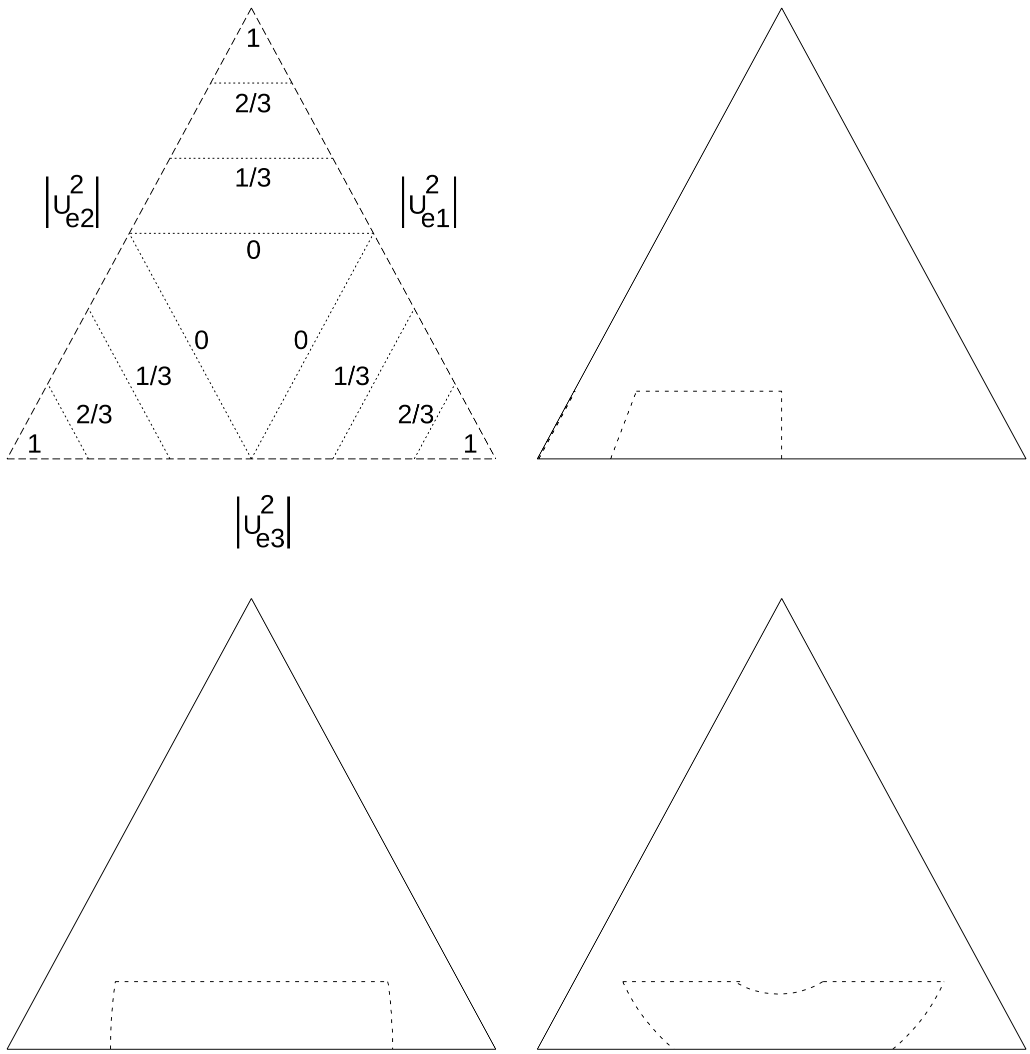

Figure 3: Case of "nearly degenerate" neutrino spectrum ($m_1 \approx m_2 \approx m_3$). From up to down, left to right: 1^{st} plot, the minimum value of $\mathcal{M}_{\rm ee}/m_1$ represented in the unitarity triangle. Reference numerical values of 1, 2/3, 1/3 and 0 are indicated. 2^{nd} plot, indicative allowed regions for MSW enhanced transitions; the SMA solution is almost superimposed to the left–second–side, where $U_{\rm e2} = 0$; 3^{rd}, allowed region for vacuum oscillations; 4^{th}, allowed region for averaged oscillations. We assume $|U^2_{\rm e3}| < 0.15$ [1] and "normal" hierarchy. "Inverted" hierarchy corresponds approximatively to a 120° rotation of the last three plots.

3.4 A complementary representation

In order to recapitulate and confirm the results obtained in this section, we present a complementary graphical representation. Supposing that the mixing elements are known with good precision, we can plot the range of values of $\mathcal{M}_{\rm ee}$ as a function of the only residual parameter: The mass of the lightest neutrino[4]. This is done in fig. 4, where we assume the mass splittings $\Delta m^2_{atm} = 2\times 10^{-3}$ eV^2 and $\Delta m^2_\odot = 10^{-4}$ eV^2 for "normal" and "inverted" hierarchy. The mixing $|U^2_{\rm e3}|$ (resp. $|U^2_{\rm e1}|$) with the heaviest (resp. lightest) state is $0, 2, 4$ and $6\ \times 10^{-2}$ in the 4 types of curves, going from inner to outer ones. We fixed $|U^2_{\rm e2}| = 0.4$ (resp. $|U^2_{\rm e3}| = 0.4$), which corresponds roughly to an LMA solution. The figure confirms the conclusions obtained in section 3.1 for the case $[\mathcal{N}]$, about the importance of $\Delta m^2_\odot$, and of the small mixing element $|U^2_{\rm e3}|$. For the case $[\mathcal{I}]$, instead, $|U^2_{\rm e1}|$ and $\Delta m^2_\odot$ are less important in agreement with the discussion in section 3.2.

This representation emphasizes that also a *null* experimental result may be a very important information on the massive neutrino parameters: In fact, $\mathcal{M}_{\rm ee}^{min} \lesssim 10^{-2}$ eV could rule out the assumption of "inverted" hierarchy, see the second plot of fig. 4; or, a bound on $\mathcal{M}_{\rm ee}^{min}$ at the 10^{-3} level could amount to a measurement of the lightest neutrino mass, see the first plot of the same figure. Unfortunately, the value of m_1 determined in this way depends strongly on the parameters of oscillation, since:

$$\mathcal{M}_{\rm ee}^{min} = \Big|\ |U^2_{\rm e2}|\ (\Delta m^2_\odot)^{1/2} - |U^2_{\rm e3}|\ (\Delta m^2_{atm})^{1/2}\ \Big| \qquad \text{for } m_1 = 0; \tag{3.7}$$

so that, even in the LMA case we are considering, it will be a real challenge to prove that $m_1 \neq 0$.

4. Concluding remarks

4.1 On the case $\mathcal{M}_{\rm ee} \approx 0$

We regarded $\mathcal{M}_{\rm ee}$ as a function of several parameters: the mixing elements, the squared mass splittings, the mass of the lightest neutrino and the complex phases. Following this approach, one may be led to wonder whether the cases when the rate is small as a consequence of cancellations among the various parameters are (in some sense) "natural".

We show here how the smallness can arise in a "natural" manner. Let us postulate that the neutrino mass matrix has a hierarchical structure, analogous to the structure of the Yukawa couplings of the charged fermions.

[4]In practice, this representation will be useful when the parameters of oscillation will be known reliably.

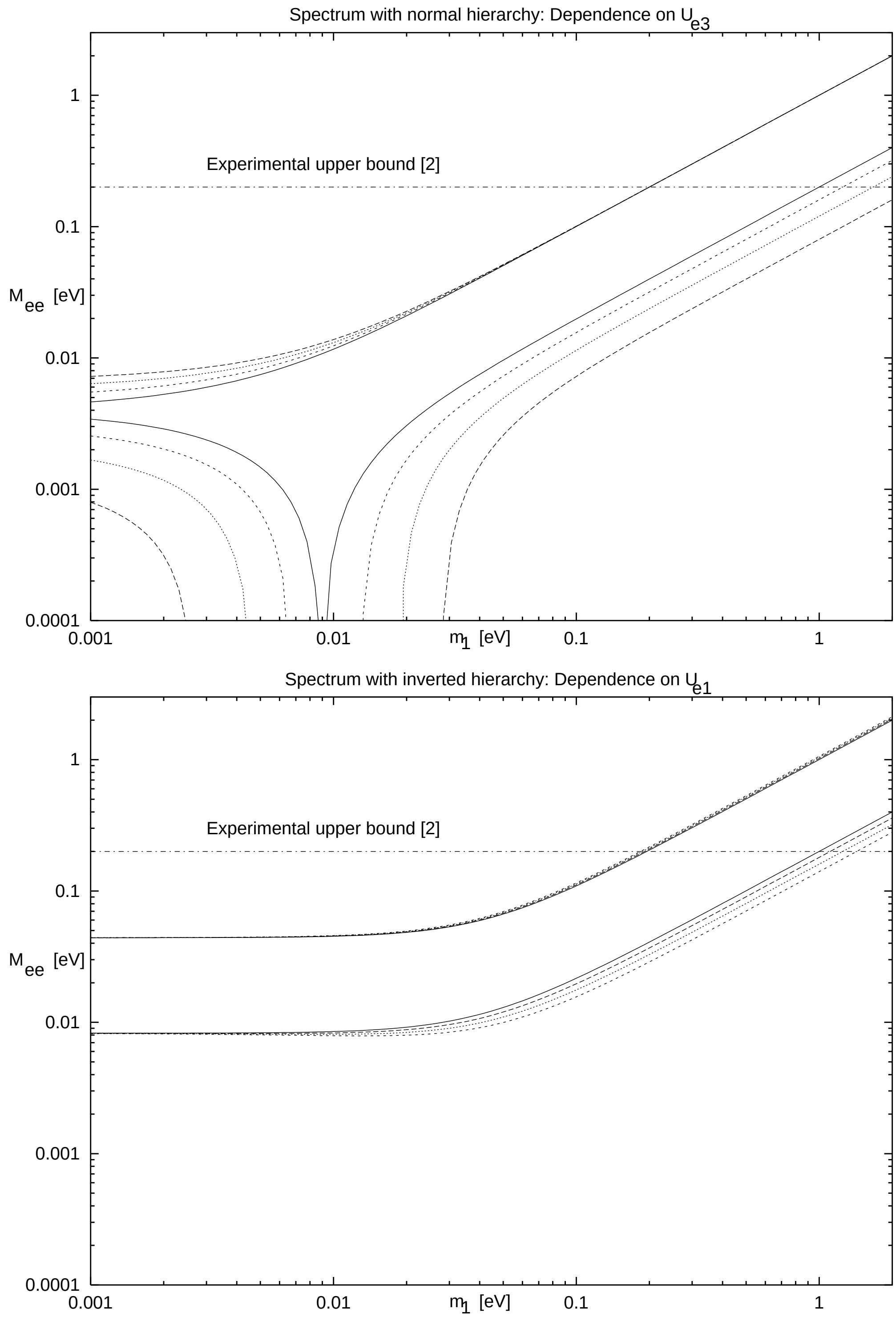

Figure 4: Range of values of $\mathcal{M}_{\rm ee}$. The four upper curves in each plot represent $\mathcal{M}_{\rm ee}^{max}$, the four lower ones $\mathcal{M}_{\rm ee}^{min}$ — eqs. (1.2), (1.3). The four types of curve differ for the values of the sub-dominant mixing element: For "normal" hierarchy, $|U_{\rm e3}^2| = 0, 2, 4$ and 6×10^{-2}, going from inner curves (continuous line) to outer ones (long-dashed); same values, but for $|U_{\rm e1}^2|$, in the case of "inverted" hierarchy.

In this case, we can expect that the "ee-entry" of the neutrino mass matrix ($= \mathcal{M}_{\rm ee}$) is the smallest one, and also $\mathcal{M}_{\rm ee} \ll (\Delta m^2_{atm})^{1/2}$. This is what happens in the two models of references [14], where:

$$\mathcal{M}_{\rm ee} \approx (\Delta m^2_{atm})^{1/2} \times (\sin\theta_C)^{2n}; \tag{4.1}$$

θ_C is the Cabibbo angle, and $n = 2, 3$ in the two models respectively. The value of $\mathcal{M}_{\rm ee}$ in these models is rather small (see also [15]). Although the contribution from third family is modest, LMA solutions with relatively large mass splittings are possible in this type of models [16], which *a priori* may imply much larger values of $\mathcal{M}_{\rm ee}$, as remarked for the case of section 3.1. Thus, these models provide examples of cases when $\mathcal{M}_{\rm ee}$ is small as a consequence of cancellations among the various contributions.

In another sense, the statement $\mathcal{M}_{\rm ee} \approx 0$ is surely "natural" in a standard model framework, since at one loop level the radiative corrections are tiny: $\sim y_e^2/(4\pi)^2 \sim 5 \times 10^{-14}$, where y_e is the electron Yukawa coupling.

4.2 What is the maximum value of $\mathcal{M}_{\rm ee}$?

Let us briefly summarize the results of section 3, about an aspect of importance for experimental search: The maximum value of $\mathcal{M}_{\rm ee}$ that we can *a priori* expect.

For given mixing elements, $\mathcal{M}_{\rm ee}^{max}$ *increases* passing from the cases discussed in sections 3.1 (case $[\mathcal{N}]$) to section 3.2 (case $[\mathcal{I}]$), and finally to section 3.3 (case $[\mathcal{D}]$). Indeed, $\mathcal{M}_{\rm ee}^{max}$ reaches at most the 10^{-2} eV level in case $[\mathcal{N}]$, depending on the sub-dominant mixing element $|U^2_{\rm e3}|$ and on the scenario of oscillation (eq. (3.2)); it can be of the order of 3 to 9×10^{-2} eV in the case $[\mathcal{I}]$, depending on the size of Δm^2_{atm} (eq. (3.5)); finally, $\mathcal{M}_{\rm ee}^{max}$ can be as large as the experimental upper limit of 0.2 eV in the case $[\mathcal{D}]$. In this sense, the *a priori* hope of a positive experimental result increases when going from $[\mathcal{N}]$ to $[\mathcal{I}]$, and from $[\mathcal{I}]$ to $[\mathcal{D}]$[5].

4.3 Studies of neutrino oscillations and search for $0\nu 2\beta$ decay

We have shown that the parameters of oscillations are strictly related to the possible value of the $0\nu 2\beta$ decay rate. However, the dependence on the type of spectrum is also essential. We summarize here some results of special interest (making reference for details to the previous section):

- For the small angle MSW solution, $\mathcal{M}_{\rm ee}$ is quite large for "inverted" hierarchy in the case $m_1 \ll (\Delta m^2_{atm})^{1/2}$, see eq. (3.6); for "normal" hierarchy, we have instead eq. (3.3), which is smaller than the previous case by a factor of $|U^2_{\rm e3}|$ when m_1 is small, and possibly even smaller (eq. (3.3)).

[5] On the contrary, one might argue that the case $[\mathcal{N}]$ is more likely than $[\mathcal{I}]$, and this latter more likely than $[\mathcal{D}]$, again on the basis of an analogy between the neutrino spectrum and the spectra of the charged fermions.

• For the large mixing angle MSW solution, contributions from "solar" frequency, order $(\Delta m^2_\odot)^{1/2}$ are *not* negligible, and they may lead to cancellations (or enhancements) depending on the size of $|U^2_{e3}|$ in the case of "normal" hierarchy (sections 3.1 and 3.4).
• For VO solution, and "normal" hierarchy, the dependence of $\mathcal{M}^{min}_{ee}$ on $|U^2_{e3}|$ in quite appreciable (section 3.1).
• For "inverted" hierarchy, cancellations are not easy to obtain if m_1 is small in comparison with $(\Delta m^2_{atm})^{1/2}$, except for solutions of the solar neutrino problem with almost maximal mixing angles (section 3.2).
• Largest values of $\mathcal{M}_{ee}$ are taken in the case of "nearly degenerate" spectrum, $m_1 \gg (\Delta m^2_{atm})^{1/2}$ (section 3.3). In this extreme case, cancellations are possible especially for quite large mixing angle solutions, with relevant dependence on the size of the sub-dominant mixing, for both "normal" and "inverted" hierarchies.

4.4 Conclusions and perspectives

In this work, we discussed the interplay between the studies of neutrino oscillations and the search for $0\nu2\beta$ decay. We introduced new graphical representations, aimed at clarifying the relations between the neutrino spectra, the scenarios of oscillations and the rate of the neutrinoless double beta decay. For the perspectives, it has to be noticed that the present information on massive neutrinos is compatible with quite different oscillations scenarios and neutrino spectra. Future experiments aiming at a signal of the $0\nu2\beta$ process above the 10^{-2} eV level [17] will have an important role in deciding among the alternative possibilities.

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